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COUPLING IMPEDANCE OF LAMINATED MAGNETS IN THE BOOSTER

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The magnets in the Fermilab Booster Synchrotron are laminated in order to minimize the eddy current losses and associated field distortions expected at a 15 Hz operating frequency. To further reduce the eddy current losses, the vacuum chamber is outside the magnet, hence allowing the beam induced wall currents to see the laminations directly. This Note estimates the coupling impedances and beam energy loss per turn caused by the exposed laminations.

1. CIRCULAR GEOMETRY

We begin by considering an ideal circular beam pipe of radius a as shown in Figure 1. A small azimuthal gap of length g where g<<a is placed in the beam pipe, and the gap couples to one end of a long cylindrical transmission line formed by placing an additional pipe of radius b outside the beam pipe. We will assume that the transmission line thus formed is sufficiently long and sufficiently lossy that there are no reflections at the far end which could propagate back to the gap. Hence the characteristic impedance of the transmission line, as seen at the gap, is (where we have chosen b-a=g):

$$Z_c = \frac{377}{2\pi} \mathrm{Ln}\left(\frac{b}{a}\right) \cong \frac{377}{2\pi a}(b-a) = \frac{377g}{2\pi a} \text{ ohms} \qquad (1.1)$$

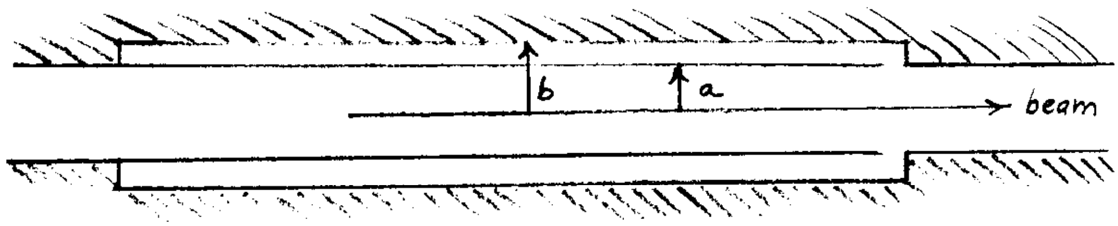

Figure 1. Beam in circular beam pipe of radius a coupled to coaxial transmission line of inner radius a and outer radius b. Beam induces TEM mode in coaxial transmission line.

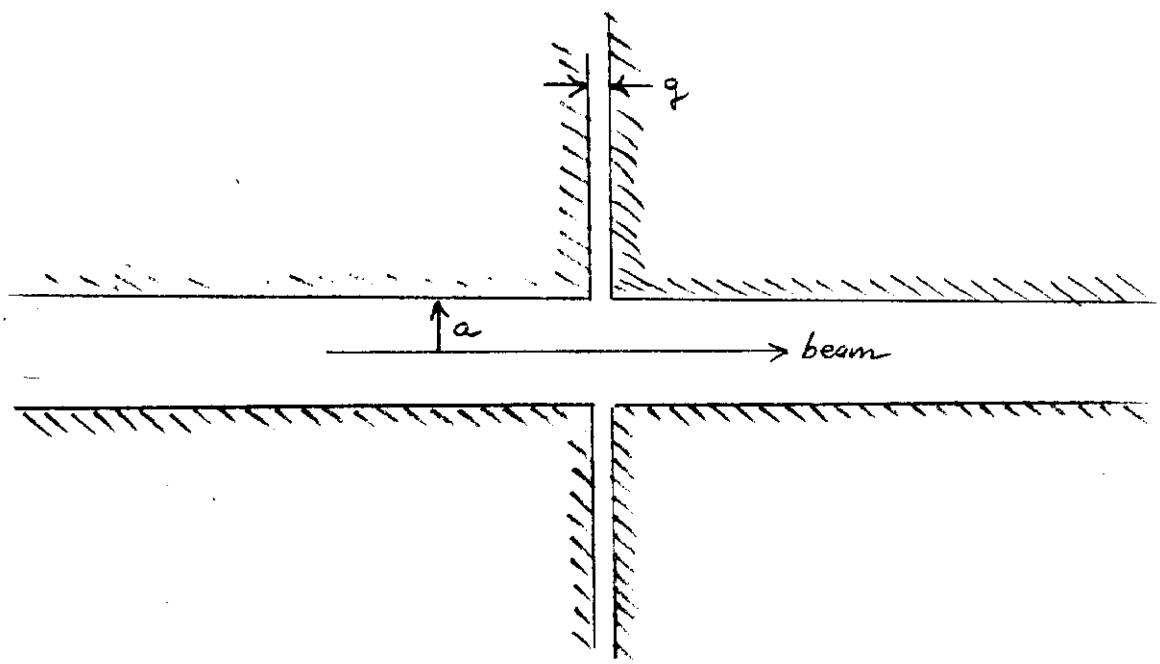

Figure 2. Beam in circular beam pipe of radius a coupled to radial transmission line of gap g. Beam induces TEM mode in radial transmission line.

If we assume that the proton beam is of sufficiently high energy that the associated fields are essentially TEM, then the distributed image currents on the inner wall of the vacuum chamber are identical in magnitude, coincident in time, and opposite in sign to the beam itself. These image currents flowing across the gap create a voltage drop

$$V(t) = -Z_c\ I_b(t) \tag{1.2}$$

which is seen by the beam as a longitudinal impedance. The real part of the gap impedance creates power losses (i.e beam energy loss per turn), and the reactive part gives rise to the longitudinal and transverse coupling impedances. If the beam is off-center, and is at a location r_o, Θ_o, then the azimuthal image current distribution is given by (all currents are rms values):

$$i(\omega, \Theta) = \frac{-I_b(\omega)}{2\pi a}\left[1 + \frac{2r_o}{a}\cos(\Theta - \Theta_o)\right] \tag{1.3}$$

Here we consider only one Fourier component of the beam current, since for complex geometries the gap impedance will be frequency dependent. The Fourier components are harmonics of the revolution frequency

$$\omega = n\omega_o = 2\pi n f_o \tag{1.4}$$

From equation 1.1 above the gap admittance per unit azimuthal width is

$$Y(\omega) = \frac{1}{377g} \tag{1.5}$$

Hence the power loss per unit azimuthal width is

$$p(\omega,\Theta) = \frac{i^2(\omega,\Theta)}{Y(\omega)} = 377g\ i^2(\omega,\Theta) \tag{1.6}$$

and the total power loss is for m gaps

$$P(\omega) = m\int_0^{2\pi} p(\omega,\Theta)\,a d\Theta = \frac{377mg}{2\pi a}\left[1 + \frac{2r_0^2}{a^2}\right] I_b^2(\omega) \text{ watts} \tag{1.7}$$

The longitudinal resistance of m gaps at frequency ω is then

$$\mathrm{Re}\ Z_L(\omega) = \frac{P(\omega)}{I_b^2(\omega)} = \frac{377mg}{2\pi a}\left[1 + \frac{2r_0^2}{a^2}\right] \text{ ohms} \tag{1.8}$$

To create the equivalent of a lamination gap, we rotate the cylindrical transmission line outward to make a radial transmission line, as shown in figure 2, thus creating a lamination gap. This gap, like the cylindrical transmission line, supports outward going TEM waves. Due to the fact that the solution for a radial line is given by Bessel functions, there is for the frequencies of interest here a small fudge factor[1]. Hence the real part of the longitudinal impedance is for m gaps per turn:

$$\mathrm{Re}\ Z_L(\omega) = 1.4\ \frac{377mg}{2\pi a}\left[1 + \frac{2r_0^2}{a^2}\right] \text{ ohms} \tag{1.9}$$

and the total impedance is

$$Z_L(\omega) = \mathrm{Re}\ Z_L(\omega) + j\mathrm{Im}\ Z_L(\omega) \tag{1.10}$$

If the imaginary part of the gap impedance is not known, or if only the real part is known from attenuation measurements, the imaginary part may be obtained from dispersion relations[2]:

$$\mathrm{Im}\ Z_L(\omega) = \frac{2\omega}{\pi}\int_0^{\infty} \frac{\mathrm{Re}\ Z_L(\omega') - \mathrm{Re}\ Z_L(\omega)}{(\omega')^2 - \omega^2}\,d\omega' \tag{1.11}$$

The conventional longitudinal and transverse coupling impedances may now be written down. The transverse coupling impedance arises from the spacial dependence of the longitudinal component.

$$\left(\frac{Z}{n}\right)_L = \frac{I_m\ Z_L(\omega)}{n} \qquad (1.12)$$

$$Z_T = \frac{2R}{a^2}\left(\frac{Z}{n}\right)_L \qquad (1.13)$$

where R is the machine radius. The energy loss per turn can now be estimated. Consider a centered parabolic beam bunch of full length at base = W, and N particles in B identical bunches. The instantaneous current is

$$I_b(t) = \frac{3eN}{2WB}\left[1 - \left(\frac{2t}{W}\right)^2\right]; \quad |t| < \frac{W}{2} \qquad (1.14)$$

The total power dissipation rate for B bunches and a revolution frequency f_o is

$$\langle P\rangle = \langle \mathrm{Re}\ Z_L(\omega)\rangle f_o B \int_{-W/2}^{W/2} I_b^2(t)\ dt$$

$$= \frac{39}{20}\ \frac{e^2N^2 f_o}{WB}\langle \mathrm{Re}\ Z_L(\omega)\rangle \text{ watts} \qquad (1.15)$$

where the part in brackets represents the longitudinal resistance averaged over the power spectrum of the beam. Dividing by the total number of particles N and the revolution frequency f_o gives the average energy loss per turn:

$$\text{Energy loss per turn} = \frac{39}{20}\ \frac{eN}{WB}\langle \mathrm{Re}\ Z_L(\omega)\rangle\ \frac{eV}{\text{turn}} \qquad (1.16)$$

2. BOOSTER LAMINATION GEOMETRY

The Fermilab Booster lamination geometry is a parallel plane geometry as shown in Figure 3, neglecting the fact that it is actually a combined function machine. For this geometry, the image current density for a centered beam is given by

$$i(x) = \frac{-I_b(\omega)}{2h} \quad \text{sech}\left(\frac{\pi x}{h}\right) \tag{2.1}$$

If we again use the lamination gap admittance per unit width in equation 1.5, the power loss for m lamination per turn gaps is (compare to equation 1.7), where y= vertical displacement of beam from the center line of the aperture:

$$P(\omega) = \frac{377mg}{\pi h}\left[1 + \frac{2\pi^2}{3} \quad \frac{y^2}{h^2}\right] I_b^2(\omega) \text{ watts} \tag{2.2}$$

resulting in a longitudinal resistance per turn of (including fudge factor):

$$\text{Re } Z_L(\omega) = \frac{P(\omega)}{I_b^2(\omega)} = 1.4 \; \frac{377mg}{\pi h}\left[1 + \frac{2\pi^2}{3} \quad \frac{y^2}{h^2}\right] \text{ohms} \tag{2.3}$$

It is implicitly assumed here that the attenuation in the lamination gap is sufficiently large that none of the outward going TEM wave is reflected back to the aperture. This is verified by wire measurements above 100 MHz. At very low frequencies, since the laminations are shorted together at the outside of the magnet, both the real and imaginary components of the gap impedance go to zero. The imaginary component rises nearly linearly with frequency (inductive, like a shorted coax), and the real part rises nearly quadratically with frequency (required by dispersion relations), since the losses are proportional to the square of the induced current in the lamination gap. The losses are due to both skin depth losses in the laminations and loss tangent in the lamination coating. The main mode of propagation is a TEM wave between the laminations, and therefore there is no low frequency cutoff due to the gap dimension.

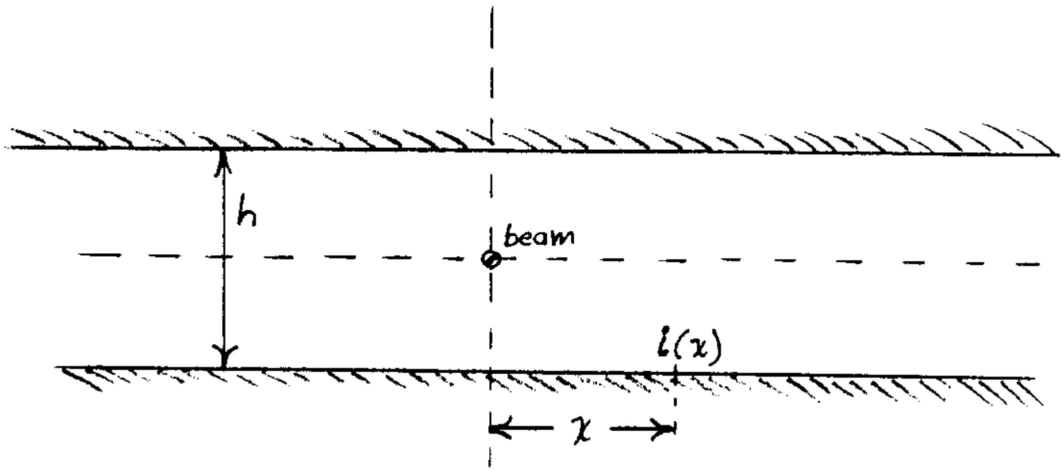

Figure 3. Booster magnet geometry - the beam is shown between two approximately parallel laminated pole tips - the induced wall current at location x is i(x).

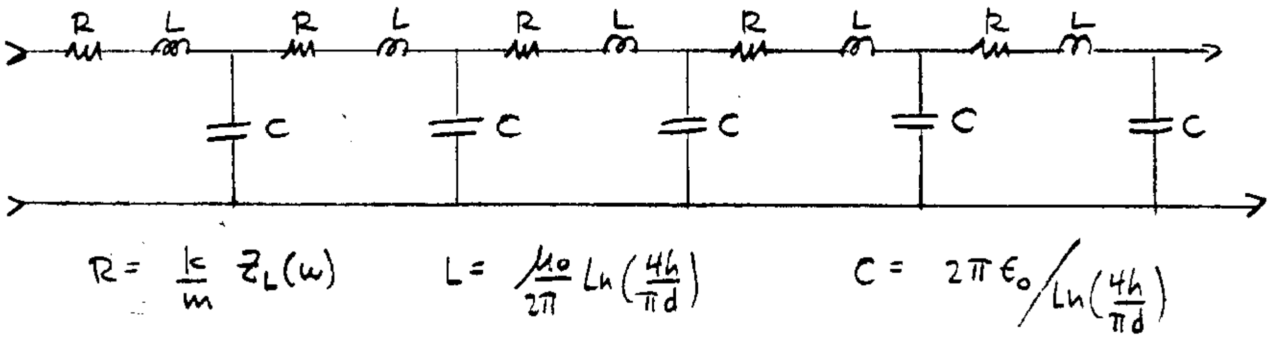

Figure 4. Equivalent circuit for Booster magnet aperture with wire in center.

3. MEASUREMENTS

We now discuss wire measurements of the lamination impedance. If a wire of diameter d is placed in the beam aperture shown in Figure 3., the characteristic impedance is, for a lossless transmission line:

$$Z_C = \sqrt{L/C} = \frac{377}{2\pi} \; Ln\left(\frac{4h}{\pi d}\right) \qquad \text{Ohms} \qquad (3.1)$$

where

$$L = \frac{\mu_o}{2\pi} \; Ln\left(\frac{4h}{\pi d}\right) \qquad \text{Henrys/m} \qquad (3.2)$$

and

$$C = 2\pi\varepsilon_o \Big/ Ln\left(\frac{4h}{\pi d}\right) \qquad \text{Farads/m} \qquad (3.3)$$

Hence if we include the lamination impedance for k gaps per meter, the equivalent circuit is shown in Figure 4. The attenuation in db/meter is then (at a sufficiently high frequency that the characteristic impedance is not significantly altered):

$$\text{Attenuation} = \frac{20}{Ln10} \; \frac{(k/m)\,\text{Re}\; Z_L(\omega)}{2Z_c} \qquad \text{db/meter} \qquad (3.4)$$

Measurements with a 1.59 mm diameter wire (characteristic impedance of 220 ohms) in actual Booster magnets indicate an attenuation of about 1.1 db/meter above 100 MHz. Hence the longitudinal impedance per meter is (k= 1600 laminations per meter, m= 460,000 laminations per turn) is

$$\frac{k}{m} \text{Re } Z_L(\omega) \cong \frac{2 \times 1.1 \times Z_c \ln 10}{20} = 55 \text{ ohms/meter} \quad (3.5)$$

The wire measurements showed a rise from zero to about 60 ohms/meter at 50MHz, dropping back down to 55 ohms/meter at about 100 MHz, and remaining at this value to 1 GHz.

For 96 three meter long magnets per turn, the expected longitudinal resistance per turn is

$$\text{Re } Z_L(\omega) = 3 \times 96 \times 55 \text{ ohms/m} = 16{,}000 \text{ ohms/turn} \quad (3.6)$$

Using $N = 1.5\times10^{12}$ protons, $B = 84$ bunches, and $W = 4.6$ nsec in equation 1.16, the calculated energy loss per turn is

$$\frac{39}{20} \frac{eN}{WB} \langle \text{Re } Z_L(\omega) \rangle = 19 \text{ KeV/turn} \quad (3.7)$$

in agreement with a measured value of 22 keV/turn.

We may use equations 2.3 and 3.6 with h=5 cm to estimate the size of the lamination gap:

$$g = \frac{\pi h \text{ Re } Z_L(\omega)}{1.4 \times 377m} = 10 \text{ microns} \quad (3.8)$$

neglecting any dielectric coating. This is reasonable. Since the laminations are about 635 microns thick, the packing fraction is about 98.4%.

4. EFFECT OF RESISTIVE LINER IN APERTURE

If the beam were shielded from the laminations with a layer of conducting material having a skin depth given by

$$\delta(\omega) = \sqrt{\frac{2\rho}{\mu_0 \omega}} \text{ meters} \tag{4.1}$$

the longitudinal resistance in 2.3 would become[3]

$$\mathrm{Re}\; Z_L(\omega) = \frac{1}{\pi h} \frac{3 \times 96}{\delta(\omega)} \rho \left[1 + \frac{2\pi^2}{3} \frac{y^2}{h^2}\right] \text{ohms} \tag{4.2}$$

Using $\omega = 2\pi \times 500$ MHz, and $\rho = 100 \times 10^{-8}$ ohm-meters, this becomes: ($h = 0.05$ meters):

$$\mathrm{Re}\; Z_L = 81 \left[1 + \frac{2\pi^2}{3} \frac{y^2}{h^2}\right] \text{ohms at 500 MHz} \tag{4.3}$$

The resistive wall longitudinal impedance would then be (for the dipoles only):

$$Z_L(\omega) = (1+j)\; 81 \left(\frac{\omega/2\pi}{500 \times 10^6}\right)^{\frac{1}{2}} \left[1 + \frac{2\pi^2}{3} \frac{y^2}{h^2}\right] \text{ohms} \tag{4.4}$$

from which the coupling impedances can easily be estimated.

If the aperture were 50% covered with longitudinal strips, equation 4.3 would quadruple to about 320 ohms at 500 MHz. This is still substantially less than the 16,000 ohms in equation 3.6.

1. S.C. Snowdon, Fermilab TM-277 (11/3/70)
A.G. Ruggiero, Fermilab FN-219 (12/15/70)
______________, Fermilab FN-220 (1/4/71)
______________, Fermilab FN-230 (5/15/71)
R. Gluckstern, Fermilab TM-1374 (11/85)

2. Morse and Feshbach, "Methods of Theoretical Physics", McGraw Hill (1953). See equation 4.2.21

3. K.Y. Ng, Particle Accelerators, Vol. 16, page 63 (1984). See equations 13,14,23, and 40.